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Design of Augmented Reality-Based Learning Media Using the TPACK Framework for Visualizing Graph Concepts in Discrete Mathematics

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Abstract: The increasing complexity of graph theory concepts in Discrete Mathematics presents significant challenges for students due to their abstract nature and the limitations of conventional two-dimensional instructional media. Augmented Reality (AR) has emerged as a promising technology for enhancing conceptual visualization; however, existing studies primarily emphasize application development and learning effectiveness, with limited attention to systematic instructional design based on pedagogical frameworks. This study aims to design an Augmented Reality-based learning medium using the Technological Pedagogical Content Knowledge (TPACK) framework to support graph concept visualization in Discrete Mathematics. The study employed the Design and Development Research (DDR) method, consisting of problem identification, literature review, needs analysis, TPACK-based instructional design, prototype design, and conceptual validation. The proposed learning media integrates technological, pedagogical, and content knowledge into a unified instructional framework. The design includes a conceptual system architecture, instructional flow, user interaction model, use case design, and user interface for AR-assisted graph visualization. The resulting conceptual framework provides structured guidance for developing interactive learning media that align technological capabilities with instructional strategies and graph-related content. This study contributes a comprehensive design framework that can serve as a reference for future development, implementation, and empirical evaluation of AR-based learning media in mathematics and computer science education. Future research should conduct expert validation, prototype implementation, and effectiveness testing to evaluate the educational impact of the proposed learning media.
Keywords: Augmented Reality; TPACK; Discrete Mathematics; Graph Visualization; Educational Technology.

INTRODUCTION

The rapid advancement of digital technologies has significantly transformed higher education by encouraging the adoption of innovative teaching approaches that enhance students' learning experiences. Among the emerging educational technologies, Augmented Reality (AR) has received considerable attention because of its capability to seamlessly integrate virtual objects into real-world environments, creating interactive and immersive learning experiences. Unlike conventional digital media, AR enables learners to visualize abstract concepts in three-dimensional environments and interact with digital objects in real time, thereby promoting active engagement and conceptual understanding. Recent studies have reported that AR positively influences students' motivation, engagement, spatial reasoning, and conceptual understanding, particularly in science, technology, engineering, and mathematics (STEM) education (Santos & Castro, 2021; Belda-Medina & Calvo-Ferrer, 2022).

In computer science education, Discrete Mathematics serves as one of the fundamental courses supporting advanced topics such as algorithms, artificial intelligence, cryptography, data structures, computer networks, and software engineering. Among its learning materials, graph theory plays a central role because it provides mathematical models for representing relationships among objects and solving various computational problems. Concepts such as vertices, edges, paths, cycles, trees, spanning trees, and graph traversal algorithms are essential

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foundations for numerous computing applications. Nevertheless, these concepts are highly abstract and are often difficult for students to comprehend through conventional instructional approaches.

Traditional learning media generally rely on textbooks, presentation slides, and whiteboard explanations, which primarily present graph structures in static two-dimensional representations. Although these approaches effectively introduce theoretical concepts, they often fail to illustrate the dynamic relationships among graph components, limiting students' ability to construct meaningful conceptual understanding. Consequently, students frequently experience difficulties in visualizing graph structures, understanding graph properties, and applying graph concepts to computational problem-solving. These challenges indicate the need for innovative instructional media capable of transforming abstract graph concepts into interactive and meaningful learning experiences.

Augmented Reality has emerged as a promising technology for addressing these challenges by providing three-dimensional visualization and real-time interaction with virtual learning objects. Through AR technology, students are able to observe graph structures from multiple perspectives, manipulate graph objects directly, and explore relationships among graph components in an immersive learning environment. Previous studies have demonstrated that AR-based learning media improve learners' engagement, visualization ability, and conceptual understanding across various educational disciplines. However, most previous studies have primarily emphasized application development or learning effectiveness while paying relatively limited attention to systematic instructional design grounded in educational theory.

To ensure effective technology integration within educational environments, the Technological Pedagogical Content Knowledge (TPACK) framework has become one of the most widely accepted theoretical models. TPACK emphasizes the integration of Technological Knowledge (TK), Pedagogical Knowledge (PK), and Content Knowledge (CK) to ensure that educational technology supports both instructional strategies and learning content. Rather than focusing solely on technological innovation, TPACK provides comprehensive guidance for designing learning environments in which technology, pedagogy, and subject matter complement one another to facilitate meaningful learning experiences.

Despite the growing number of studies concerning Augmented Reality and TPACK, several important research gaps remain. First, previous AR-based educational studies have largely focused on developing learning applications and evaluating learning outcomes, whereas relatively few studies have proposed systematic instructional design frameworks that explicitly integrate technological, pedagogical, and content knowledge. Second, studies implementing the TPACK framework have predominantly investigated teachers' competencies in technology integration rather than its application in designing AR-based learning media for specific instructional content. Third, although AR has been widely applied in mathematics education, existing research mainly focuses on geometry, spatial visualization, and science learning, while studies specifically addressing graph visualization in Discrete Mathematics remain limited. Consequently, there remains a need for a comprehensive instructional design framework capable of systematically integrating AR technology with TPACK principles to support graph concept visualization in higher education.

The novelty of this study lies in the development of a conceptual instructional design framework that systematically integrates Augmented Reality technology with the TPACK framework specifically for graph concept visualization in Discrete Mathematics. Unlike previous studies that primarily focused either on AR application development or on the implementation of TPACK independently, the proposed framework explicitly aligns technological capabilities, pedagogical strategies, and graph-related content within a unified instructional design. Furthermore, the framework provides practical guidance for designing AR-based learning media that can support future implementation, empirical evaluation, and further development in mathematics and computer science education.

Therefore, this study aims to design an Augmented Reality-based learning medium using the TPACK framework for graph concept visualization in the Discrete Mathematics course. The proposed design is expected to provide a systematic reference for developing technology-enhanced learning media that integrate technological innovation, pedagogical effectiveness, and subject content while serving as a conceptual foundation for future implementation and effectiveness studies.

LITERATURE REVIEW

Learning Media

Learning media refers to all tools, resources, and technologies utilized to facilitate the delivery of instructional content and support the achievement of learning objectives. In modern educational environments, learning media plays a crucial role in enhancing student engagement, motivation, and knowledge acquisition. The integration of digital technologies into education has transformed traditional learning environments into interactive and student-centered experiences (I. M. B. Tarigan, 2026).

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According to constructivist learning theory, knowledge is actively constructed by learners through interaction with their environment and learning experiences. Therefore, instructional media should not merely serve as a channel for information delivery but also provide opportunities for exploration, interaction, and reflection (I. M. Tarigan et al., 2022). Interactive learning media can promote meaningful learning by enabling students to actively engage with content and construct their own understanding.

In higher education, particularly in science, technology, engineering, and mathematics (STEM) disciplines, learning media must be capable of representing complex and abstract concepts in a manner that is accessible and understandable. The use of innovative technologies such as Augmented Reality has emerged as a promising approach to address these challenges by providing immersive and interactive learning experiences.

Augmented Reality in Education

Augmented Reality (AR) is a technology that overlays virtual objects, information, or multimedia elements onto the real-world environment in real time. Unlike Virtual Reality (VR), which immerses users in a completely virtual environment, AR enhances the real world by integrating digital content while maintaining users' awareness of their physical surroundings (Lachner et al., 2021).

According to Ronald Azuma, AR systems possess three essential characteristics:

a. Integration of virtual and real-world elements.
b. Real-time interaction.
c. Three-dimensional registration of virtual objects.

These characteristics make AR particularly suitable for educational applications because they allow learners to interact directly with virtual representations of concepts that may otherwise be difficult to observe or understand.

In educational settings, AR has demonstrated significant potential in improving learning outcomes by:

a. Increasing learner motivation and engagement.
b. Supporting experiential and exploratory learning.
c. Enhancing spatial visualization skills.
d. Facilitating conceptual understanding.
e. Providing contextualized learning experiences.

Several studies have reported positive effects of AR on student learning performance. AR-based learning environments enable learners to manipulate virtual objects, observe dynamic processes, and receive immediate visual feedback. Such features are especially valuable in subjects involving abstract structures, spatial reasoning, and complex relationships (Putri et al., 2022).

Within mathematics education, AR can bridge the gap between abstract mathematical representations and concrete visual experiences. Students can visualize mathematical structures in three-dimensional forms, making it easier to understand relationships, patterns, and underlying principles.

Technological Pedagogical Content Knowledge (TPACK)

The Technological Pedagogical Content Knowledge (TPACK) framework was introduced by Punya Mishra and Matthew J. Koehler as an extension of pedagogical content knowledge theory. TPACK provides a comprehensive framework for understanding how technology can be effectively integrated into teaching and learning processes (Chen & Wong, 2026).

The framework emphasizes that successful technology integration requires the interaction of three primary forms of knowledge:

Content Knowledge (CK)

Content Knowledge refers to a teacher's understanding of the subject matter being taught. In the context of this study, CK encompasses knowledge of Discrete Mathematics, particularly graph theory concepts such as vertices, edges, paths, cycles, spanning trees, and graph traversal algorithms.

A strong foundation in content knowledge is essential because inaccurate or incomplete representations of mathematical concepts may lead to misconceptions among students.

Pedagogical Knowledge (PK)

Pedagogical Knowledge refers to understanding instructional methods, learning theories, classroom management strategies, and assessment techniques.

For graph theory learning, pedagogical approaches may include:

a). Inquiry-based learning
b). Problem-based learning
c). Visual learning

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d). Collaborative learning
e). Exploratory learning
These approaches encourage active student participation and facilitate deeper conceptual understanding.

Technological Knowledge (TK)

Technological Knowledge involves understanding how to use digital tools and technologies to support learning.

In this research, TK is represented by Augmented Reality technology, including:
a). AR visualization techniques
b). Three-dimensional modeling
c). Interactive object manipulation
d). Mobile-based AR applications

The selection of AR technology is based on its capability to represent abstract graph structures in a visually engaging manner.

Intersections within the TPACK Framework

The interaction among CK, PK, and TK creates three intermediate knowledge domains (Choi & Young, 2021):
a). Pedagogical Content Knowledge (PCK)
PCK represents the understanding of how specific content can be effectively taught.
In graph theory education, PCK involves selecting instructional strategies that facilitate students' understanding of graph structures and relationships.
b). Technological Content Knowledge (TCK)
TCK refers to understanding how technology can represent and transform content.
In this study, TCK is reflected in the use of AR to visualize graph structures that are traditionally represented through static diagrams.
c). Technological Pedagogical Knowledge (TPK)
TPK focuses on understanding how technology influences teaching methods and learning processes. AR technology supports active learning by enabling students to explore graph concepts through direct interaction with virtual objects (Kim & Ok, 2021). The integration of these components forms TPACK, which serves as the theoretical foundation for designing effective technology-enhanced learning environments.

Discrete Mathematics

Discrete Mathematics is a branch of mathematics that studies discrete structures rather than continuous quantities. It forms the mathematical foundation of computer science and is widely applied in software engineering, data science, artificial intelligence, cybersecurity, and networking (Nilsson, 2024; Nusroh et al., 2022).

Unlike continuous mathematics, which deals with infinitely divisible quantities, discrete mathematics focuses on countable and distinct objects.

Major topics in Discrete Mathematics include:
a. Mathematical Logic
b. Set Theory
c. Relations and Functions
d. Combinatorics
e. Graph Theory
f. Trees
g. Recurrence Relations
h. Boolean Algebra

These topics are fundamental to computational thinking and algorithmic problem-solving. However, many concepts in Discrete Mathematics are highly abstract and require strong visualization skills. Students frequently struggle to understand relationships among discrete structures due to the lack of effective visual representations.

Augmented Reality-Based Learning Media Design Using the TPACK Approach

The proposed learning media design integrates the three dimensions of TPACK to create an effective learning environment for graph theory education (Irwanto, 2021; Prihantini et al., 2026).
a. Technological Dimension (TK)

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The technological component focuses on the use of AR technology to generate interactive three-dimensional graph visualizations. Students can observe graph structures from multiple perspectives and interact with virtual objects through mobile devices.

b. Pedagogical Dimension (PK)

The pedagogical component emphasizes active and exploratory learning. Students are encouraged to investigate graph structures, identify relationships among vertices, and solve graph-based problems through interaction with AR models.

c. Content Dimension (CK)

The content component includes fundamental graph theory concepts such as:

a). Vertices and Edges
b). Degree of a Vertex
c). Paths and Circuits
d). Trees
e). Spanning Trees
f). Graph Traversal Concepts

The integration of these dimensions ensures that technology supports pedagogical objectives while accurately representing mathematical content.

METHOD

This study employed the Design and Development Research (DDR) method to design an Augmented Reality (AR)-based learning medium using the Technological Pedagogical Content Knowledge (TPACK) framework for graph concept visualization in the Discrete Mathematics course. The DDR approach was selected because the primary objective of this study was to develop a conceptual learning media design rather than to implement and evaluate its effectiveness (Yanuarto & Iqbal, 2022). The research consisted of six sequential stages: (1) problem identification, (2) literature review, (3) needs analysis, (4) TPACK-based instructional design, (5) conceptual prototype development, and (6) expert validation and design refinement.

The research began with problem identification to examine learning difficulties encountered by students in understanding graph theory concepts in the Discrete Mathematics course. Graph theory is considered one of the most abstract topics because students frequently experience difficulties in understanding the relationships among vertices, edges, paths, cycles, trees, and spanning trees using conventional two-dimensional learning media such as textbooks, whiteboards, and presentation slides. These findings indicated the need for a learning medium capable of providing more interactive and meaningful visualization of graph concepts.

A comprehensive literature review was subsequently conducted to establish the theoretical foundation of the proposed learning media. The review examined scientific journal articles, conference proceedings, books, and previous studies related to Augmented Reality, the TPACK framework, instructional media, graph visualization, and Discrete Mathematics education. The literature review was also used to identify current research trends, research gaps, and instructional requirements that served as the basis for developing the proposed learning media.

The needs analysis aimed to identify user requirements, instructional requirements, and system requirements for the proposed learning media. The analysis focused on identifying students' learning difficulties in graph theory, their need for interactive three-dimensional visualization, and the instructional requirements necessary to facilitate conceptual understanding. In addition, curriculum analysis was conducted to determine the learning outcomes and the scope of graph theory materials to be incorporated into the learning media, including graph concepts, graph types, paths, cycles, trees, and spanning trees. The needs analysis was conducted through a document-based analysis consisting of curriculum review and an extensive literature review. The curriculum review was used to identify course learning outcomes, instructional objectives, and the scope of graph theory materials, while the literature review examined common learning difficulties, limitations of existing instructional media, and the educational potential of Augmented Reality for supporting graph visualization. The findings from both analyses were synthesized using descriptive analysis to formulate the instructional requirements, system requirements, functional specifications, and learning scenarios of the proposed learning media.

Following the needs analysis, the learning media was designed based on the Technological Pedagogical Content Knowledge (TPACK) framework to ensure the systematic integration of technology, pedagogy, and content. The Technological Knowledge (TK) component focused on the utilization of Augmented Reality technology to provide interactive three-dimensional visualization of graph structures (Santos & Castro, 2021). The Pedagogical Knowledge (PK) component emphasized active learning, inquiry-based learning, visual learning, and exploratory learning strategies to encourage students to actively construct their understanding of graph concepts. The Content Knowledge (CK) component focused on the fundamental concepts of graph theory, including vertices,

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edges, graph types, paths, cycles, trees, and spanning trees. The integration of these three components produced an instructional design that systematically aligned technological features with pedagogical strategies and learning content.

Based on the results of the needs analysis and TPACK analysis, a conceptual prototype of the learning media was developed. The prototype design included instructional content organization, learning scenarios, media usage flow, system architecture, and user interface design. The proposed learning media consists of several integrated modules, including learning materials, AR-based graph visualization, interactive exercises, evaluation activities, and user guidance. The prototype was developed at the conceptual design level to illustrate the instructional framework before software implementation. The design process involved organizing learning content, defining user interaction flow, constructing the system architecture, developing use case scenarios, and designing user interface mock-ups. Each design component was iteratively refined to ensure consistency between technological features, pedagogical strategies, and graph learning content based on the TPACK framework. The system architecture illustrates the interaction among users, the mobile application, the Augmented Reality module, and the learning content database. Furthermore, use case diagrams were developed to describe user interaction with the system, while the user interface design included a splash screen, main menu, learning materials page, AR visualization page, exercises, evaluation, and user guide. The interface was designed by considering usability principles, consistency, and ease of navigation to provide an effective learning experience.

The conceptual prototype was subsequently prepared for expert validation to evaluate its feasibility from technological, pedagogical, and content perspectives. Design refinement framework was designed to assess the accuracy of graph theory content, instructional design, user interface, navigation structure, usability, integration of Augmented Reality technology, and the implementation of the TPACK framework within the proposed learning media. Feedback obtained during the validation process is intended to support future refinement of the conceptual design before prototype implementation and effectiveness testing.

The outputs of this study include a conceptual framework for AR-based learning media using the TPACK approach, instructional design, system architecture, user interface design, and a conceptual prototype for graph concept visualization in the Discrete Mathematics course. These outputs are expected to provide a reference for future development, implementation, and empirical evaluation of AR-based learning media in mathematics and computer science education. The overall research procedure is illustrated in Figure 1.

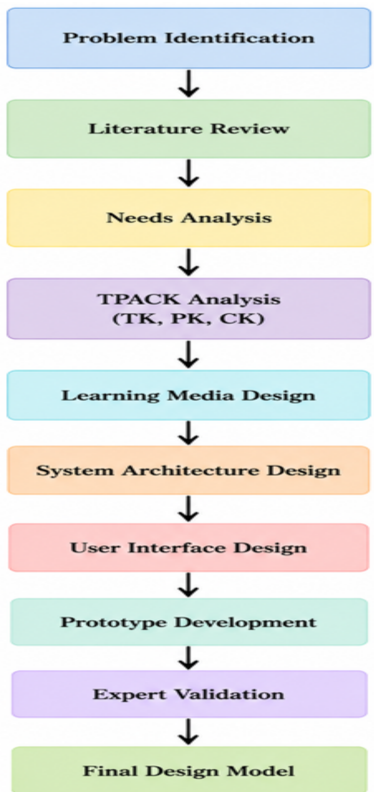

Fig. 1 Research Framework for Designing Augmented Reality-Based Learning Media Using the TPACK Approach for Graph Concept Visualization in Discrete Mathematics

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RESULT

Findings from the Needs Analysis

The needs analysis identified several instructional and technological requirements for developing an Augmented Reality (AR)-based learning medium to support graph concept visualization in the Discrete Mathematics course. The analysis revealed that graph theory remains one of the most challenging topics because many of its concepts, such as vertices, edges, paths, cycles, trees, and spanning trees, are abstract and difficult to visualize using conventional instructional media. Existing learning resources, including textbooks, lecture slides, and whiteboard illustrations, primarily present graph structures in two-dimensional representations, limiting students' ability to understand the relationships among graph components.

Furthermore, the literature review indicated that although AR has been widely implemented in mathematics and science education, most existing studies primarily emphasize technological implementation or learning effectiveness rather than instructional design. Only a limited number of studies have systematically integrated AR technology with pedagogical frameworks such as Technological Pedagogical Content Knowledge (TPACK). Consequently, the findings of the needs analysis confirmed the necessity of developing a learning media design that simultaneously integrates technological capabilities, instructional strategies, and graph learning content into a unified conceptual framework.

The outcomes of the needs analysis served as the basis for determining the instructional objectives, learning scenarios, system functionality, visualization features, and user interaction mechanisms incorporated into the proposed learning media.

TPACK-Based Learning Media Framework

Based on the identified instructional requirements, a conceptual learning media design was developed using the TPACK framework. The proposed design emphasizes the integration of Technological Knowledge (TK), Pedagogical Knowledge (PK), and Content Knowledge (CK) to ensure that the learning media functions not only as a technological innovation but also as an effective instructional tool.

The Technological Knowledge component is represented by the implementation of Augmented Reality technology, which enables students to visualize graph structures as interactive three-dimensional objects. Unlike conventional two-dimensional illustrations, AR visualization allows students to observe graph objects from multiple perspectives and interact directly with virtual representations.

The Pedagogical Knowledge component adopts active learning, inquiry-based learning, visual learning, and exploratory learning strategies. These instructional approaches encourage students to actively construct conceptual understanding by exploring graph structures independently rather than passively receiving information from lectures.

The Content Knowledge component includes fundamental graph theory materials, namely graph definitions, graph types, vertices, edges, paths, cycles, trees, and spanning trees. These concepts were selected because they represent the core learning outcomes of the Discrete Mathematics course and frequently present conceptual difficulties for undergraduate students.

The integration of these three knowledge domains resulted in a conceptual instructional framework that aligns learning objectives, instructional activities, technological features, and learning content within a single learning environment. The proposed framework is illustrated in Figure 2.

Fig. 2 Splash Screen Interface

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System Architecture Design

The proposed system architecture was developed to describe the interaction among users, the mobile application, the Augmented Reality module, and the learning content database. The architecture demonstrates how instructional content is delivered through a mobile learning environment supported by AR visualization.

Students first access learning materials through the application before activating the AR visualization module by scanning predefined markers. Once a marker is recognized, the AR engine generates corresponding three-dimensional graph objects that can be explored interactively. The learning content database stores instructional materials, graph models, evaluation items, and multimedia resources required during the learning process.

The architecture was designed to ensure that technological implementation remains consistent with instructional objectives while supporting intuitive interaction between learners and learning content. The complete system architecture is presented in Figure 3.

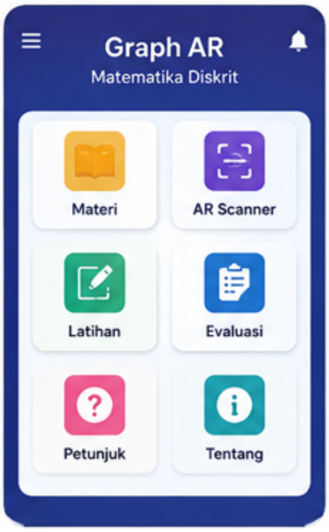

Fig. 3 Main Menu Interface

Use Case Analysis

The use case diagram illustrates the interaction between users and the proposed learning media. The design identifies students as the primary users who can access learning materials, activate AR visualization, manipulate three-dimensional graph objects, complete interactive exercises, perform learning evaluations, and access user guidance.

The interaction flow was designed based on learner-centered principles to minimize navigation complexity while maximizing opportunities for independent exploration. Each user activity corresponds to a specific instructional objective, ensuring that technology functions as a facilitator of conceptual learning rather than merely a visualization tool. The proposed use case model is presented in Figure 4.

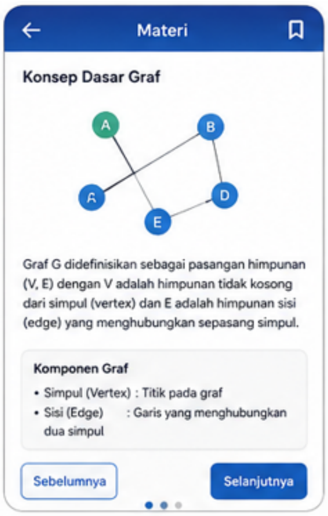

Fig. 4 Learning Materials Interface

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User Interface Design

The user interface was designed by emphasizing usability, interface consistency, simplicity, and ease of navigation. The interface consists of several interconnected modules that support the sequential learning process. The Splash Screen serves as the application's introductory interface and establishes the visual identity of the learning media before users enter the main application.

The Main Menu provides centralized navigation to all learning modules, including learning materials, AR visualization, exercises, evaluation, and user guidance. The arrangement of these menus follows the instructional sequence developed within the TPACK framework.The Learning Materials interface presents theoretical explanations of graph concepts that prepare students before interacting with AR visualization.

The AR Visualization interface functions as the core component of the learning media by enabling students to observe graph structures in three-dimensional space. Through real-time interaction, students can examine relationships among graph components more intuitively than through conventional instructional media. The Exercises module provides opportunities for students to reinforce conceptual understanding through interactive learning activities, while the Evaluation module enables students to assess their comprehension after completing the learning activities. Finally, the User Guide provides operational instructions to ensure that students can effectively utilize all application features. The complete user interface design is illustrated in Figures 5–9.

Fig. 5 AR Scanner Interface

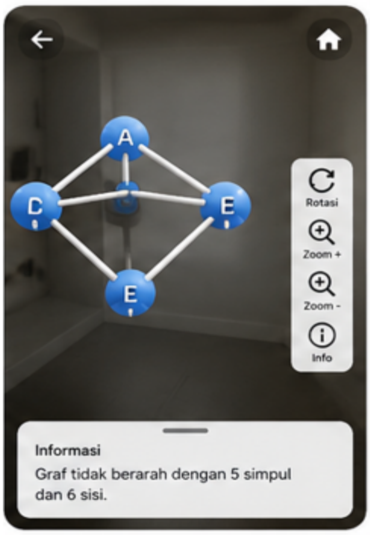

Fig. 6 3D AR Visualization Interface

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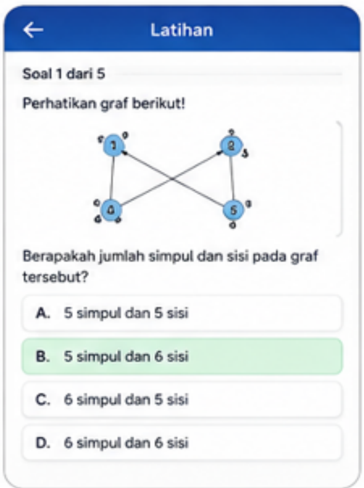

Fig. 7 Exercise Interface

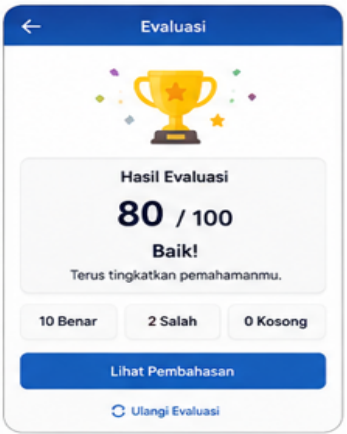

Fig. 8 Evaluation Interface

Fig. 9 User Guide Interface

DISCUSSIONS

The present study proposes a conceptual design of an Augmented Reality (AR)-based learning medium integrated with the Technological Pedagogical Content Knowledge (TPACK) framework to support graph concept visualization in the Discrete Mathematics course. The design was developed in response to the instructional challenges identified during the needs analysis, particularly the difficulty students experience in understanding

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abstract graph concepts such as vertices, edges, paths, cycles, trees, and spanning trees through conventional two-dimensional instructional media. Rather than merely introducing AR as a visualization technology, this study emphasizes the systematic integration of technological, pedagogical, and content dimensions into a coherent instructional framework.

The proposed conceptual framework provides an initial foundation for future research on technology-enhanced learning in Discrete Mathematics. By systematically integrating AR technology with the TPACK framework, the study contributes an instructional design perspective that complements existing implementation-oriented studies. This contribution is expected to facilitate future software development, instructional innovation, and empirical evaluation in higher education.

An important contribution of this study is the integration of the Technological Pedagogical Content Knowledge (TPACK) framework into the instructional design process. The proposed design demonstrates that successful integration of educational technology requires more than the inclusion of innovative digital tools. Instead, technology should be systematically aligned with appropriate pedagogical strategies and disciplinary content to create meaningful learning experiences. In the proposed framework, the pedagogical component emphasizes visual learning, inquiry-based learning, and exploratory learning approaches that encourage students to actively construct conceptual understanding through interaction with graph representations (Yang et al., 2022; Yeh et al., 2021). Simultaneously, the content component focuses on fundamental graph theory concepts that constitute essential learning outcomes in Discrete Mathematics and computer science education.

Compared with previous studies, the present research offers a different perspective on the development of AR-based learning media. Earlier studies have primarily concentrated on developing AR applications and evaluating their effectiveness after implementation (Schwajda et al., 2023; Sriadhi et al., 2022). Although these studies demonstrated the educational potential of AR, relatively limited attention has been given to the systematic instructional design process that integrates technological capabilities with pedagogical principles and subject content. Furthermore, many previous AR applications were developed for science, physics, or geometry learning, whereas research specifically addressing graph concept visualization in Discrete Mathematics remains limited. The proposed conceptual framework therefore contributes by positioning instructional design as the central element of AR integration rather than focusing solely on software development or learning outcomes.

The novelty of this study lies in the development of a conceptual instructional framework that integrates Augmented Reality technology and the TPACK model specifically for graph concept visualization in Discrete Mathematics. Unlike previous studies that either emphasized AR visualization without an explicit pedagogical framework or applied TPACK in broader technology-enhanced learning contexts, the present study systematically aligns technological features, pedagogical strategies, instructional activities, and graph learning content within a single design framework. This integrated approach provides a structured reference that can guide future development of AR-based learning media in mathematics and computer science education.

From a theoretical perspective, this study contributes to the growing literature on technology-enhanced learning by demonstrating how the TPACK framework can be operationalized in the conceptual design of AR-based instructional media. Rather than treating technology as an independent instructional element, the proposed framework highlights the importance of balancing technological innovation with pedagogical effectiveness and content representation. From a practical perspective, the design may serve as a reference for lecturers, instructional designers, and educational technology developers seeking to develop interactive learning media for abstract mathematical concepts in higher education.

Nevertheless, several limitations should be acknowledged. This research was conducted only at the conceptual design stage within the Design and Development Research (DDR) framework. Consequently, the proposed learning media have not yet undergone software implementation, expert validation, usability evaluation, or effectiveness testing in real classroom settings. Therefore, the findings should be interpreted as a conceptual contribution rather than empirical evidence of educational effectiveness. Future research should focus on prototype development, expert validation, classroom implementation, and experimental evaluation to examine the effectiveness of the proposed learning media in improving students' conceptual understanding, learning engagement, and graph visualization skills.

CONCLUSION

This study proposed a conceptual design of an Augmented Reality (AR)-based learning medium using the Technological Pedagogical Content Knowledge (TPACK) framework to support graph concept visualization in the Discrete Mathematics course. The proposed design systematically integrates technological features, pedagogical strategies, and graph learning content into a unified instructional framework, providing a structured reference for the development of technology-enhanced learning media in higher education.

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The main contribution of this study lies in the development of a conceptual instructional framework that aligns Augmented Reality technology with the TPACK model rather than focusing solely on technological implementation. The proposed design includes instructional organization, system architecture, user interaction, and interface design that collectively establish a foundation for future AR-based learning media development in mathematics and computer science education.

Nevertheless, this study is limited to the conceptual design stage of the Design and Development Research (DDR) process. The proposed learning media has not yet been implemented in classroom settings, and no expert validation, usability evaluation, or effectiveness testing has been conducted. Therefore, the findings should be interpreted as a conceptual contribution that provides guidance for subsequent development rather than empirical evidence of educational effectiveness.

Future research should focus on developing a functional prototype based on the proposed design, followed by expert validation, usability testing, and classroom implementation involving lecturers and students. Furthermore, experimental or quasi-experimental studies are recommended to investigate the effectiveness of the proposed learning media in improving conceptual understanding, learning engagement, spatial reasoning, and learning outcomes in Discrete Mathematics.

STATEMENT ON THE USE OF GENERATIVE AI

The authors acknowledge the use of AI-assisted tools (Grammarly, Mendeley, and SciteAI) exclusively for language translation, grammatical refinement, and literature search. No AI tools were used to generate the scientific concepts, data analysis, or the manuscript's original content. The authors accept full responsibility for the accuracy and integrity of the final work.

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